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Prediction of Hot Tearing Using a Dimensionless Niyama Criterion

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The dimensionless form of the well-known Niyama criterion is extended to include the effect of applied strain. Under applied tensile strain, the pressure drop in the mushy zone is enhanced and pores grow beyond typical shrinkage porosity without deformation. This porosity growth can be expected to align perpendicular to the applied strain and to contribute to hot tearing. A model to capture this coupled effect of solidification shrinkage and applied strain on the mushy zone is derived. The dimensionless Niyama criterion can be used to determine the critical liquid fraction value below which porosity forms. This critical value is a function of alloy properties, solidification conditions, and strain rate. Once a dimensionless Niyama criterion value is obtained from thermal and mechanical simulation results, the corresponding shrinkage and deformation pore volume fractions can be calculated. The novelty of the proposed method lies in using the critical liquid fraction at the critical pressure drop within the mushy zone to determine the onset of hot tearing. The magnitude of pore growth due to shrinkage and deformation is plotted as a function of the dimensionless Niyama criterion for an Al-Cu alloy as an example. Furthermore, a typical hot tear "lambda"-shaped curve showing deformation pore volume as a function of alloy content is produced for two Niyama criterion values.

INTRODUCTION

Hot tearing can be observed as cracking that forms during solidification of metal castings. Although hot tearing has been recognized in the literature for over 100 years, many questions remain about the nature and characteristics of this solidification defect. Experiments have characterized hot tearing across various alloy compositions as well as solidification circumstances such as grain refinement, superheat, and geometry.[1–3] Typically, these studies have focused on the morphological differences in the cracking and dendrite bridging at the last stages of solidification.[4] The results of these traditional hot tearing experiments can be used to produce a Λ-shaped curve with respect to alloy composition. In this case, the maximum hot tearing potential occurs at an intermediate composition usually between the pure and the eutectic composition. After comparing many of these tests, Sigworth[5] noted that the alloy composition at the maximum hot tear potential is not consistent among various hot tearing experimental geometries. More recently, experiments relating hot tearing to the high-temperature stress and the evolution of the solid fraction have been conducted.[6–8] From observations, multiple criteria for hot tearing have been developed based on stress, strain, strain rate, or other solidification parameters.[9] One prominent criterion developed by Rappaz et al.,[10] known as the RDG criterion, is based on the strain rate. The RDG criterion finds the maximum strain rate required to initiate porosity in the mushy zone. Comparing the RDG criterion to previous experimental results for hot tearing, parameters can be chosen to give a typical Λ-shaped curve. This straightforward approach models the coupled phenomena of solidification shrinkage and deformation-related effects on the pressure drop in the mushy zone for a one-dimensional situation. Once a critical pressure drop is found in the mushy zone, hot tearing is expected to occur. The critical pressure drop is the pressure drop required to nucleate porosity and in their paper was associated with earlier experimental measurements. However, the limitation of this approach becomes clear for low thermal gradients because the RDG criterion will predict hot tearing for no imposed strain rate. In this case, shrinkage porosity is expected to form and not hot tears. Instead of calculating the pressure drop across the entire mushy zone as proposed in the RDG criterion, another interpretation of these equations could be to determine the liquid fraction at the critical pressure drop, as proposed in the recently developed dimensionless Niyama approach.[11] This critical pressure drop, used as an adjustable constant, is essentially a marker for where liquid flow stops into the mushy zone. Further solidification after liquid flow ceases leads to both shrinkage porosity formation as well as porosity growth for imposed strain rates leading to hot tearing. Therefore, the purpose of the present investigation is to develop expressions to calculate both hot tearing as well as shrinkage porosity. Using prior developments for the pressure drop in the mushy zone and solving for the critical liquid fraction when flow stops with imposed strain is novel for this approach. Current developments in calculating the shrinkage porosity using the dimensionless Niyama can be employed to this end and are reviewed below.

In the metal casting industry, the Niyama criterion is regarded as the standard measure of the potential for forming feeding-related shrinkage porosity. All metal casting process simulations report this quantity as a result. Foundries rely on the criterion to diagnose local thermal gradients that are more likely to form unacceptable levels of shrinkage porosity. The Niyama criterion is defined as

$$\mathrm{Ny} = G \Big/ \sqrt{\dot{T}} \tag{1}$$

where G is the thermal gradient and $\dot{T}$ is the cooling rate, which are evaluated at a certain critical temperature during solidification. This critical temperature is usually taken to be close to, but above, the temperature at which the alloy is fully solidified. The physical basis for this relationship between thermal parameters and the shrinkage porosity is the liquid pressure drop in the mushy zone. Using a one-dimensional form of Darcy's law, this relationship has been derived by Niyama et al.[12] Recently, the Niyama criterion has been directly linked to the formation of a volume fraction of porosity for any alloy given a temperature-dependent solidification path using the dimensionless Niyama criterion or Ny^*.[11] Crucial to the development of this form of the criterion is the single-valued functional dependence of Ny^* on the critical liquid fraction, $g_{\mathrm{l,cr}}$, which is the liquid fraction when the pressure drop in the mushy zone exceeds the minimum pressure for pore nucleation. The expected amount of shrinkage porosity can be calculated after this critical liquid fraction has been reached. However, the pressure drop that leads to nucleation of porosity may occur in the case of either shrinkage or deformation. In a coupled scenario where both hot tears and shrinkage porosity may form, a contribution due to shrinkage and deformation both lead to the growth of porosity. Therefore, the expressions proposed in the RDG criterion can be revisited with the dimensionless Niyama approach and solved for the liquid fraction when the critical pressure drop is reached rather than for the maximum allowable strain rate. Additional shrinkage after this liquid fraction is expected to lead to shrinkage porosity while additional deformation is expected to lead to porosity growth that can be attributed to hot tearing.

METHOD

The representative solidification condition in the mushy zone is shown in Fig. 1 as a schematic. Conservation of mass for a one-dimensional solidification situation is used with the following assumptions. The density of the liquid, ρ_{l}, and the solid, ρ_{s}, are not equal but constant during solidification and can be expressed as total solidification shrinkage, $\beta = (\rho_{\mathrm{s}} - \rho_{\mathrm{l}})/\rho_{\mathrm{l}}$. While flow is considered one-dimensional in the spatial x direction aligned with the thermal gradient, the solid is deforming in the perpendicular direction with a constant strain rate, $\dot{\varepsilon}$. Mass conservation can be integrated moving with the constant solidification isotherm velocity of $\dot{T}\Big/G$, where $\dot{T}(= \mathrm{d}T/\mathrm{d}t)$ is the cooling rate and $G(= \mathrm{d}T/\mathrm{d}x)$ is the temperature gradient, both of which are constant. From Fig. 1, the isotherm velocity proceeds in the negative x direction and the liquid velocity is opposite in the positive direction. The final expression for the liquid velocity from mass conservation is

$$g_{\mathrm{l}}u_{\mathrm{l}} = -g_{\mathrm{l}}\beta\frac{\dot{T}}{G} - (1+\beta)\frac{\dot{\varepsilon}}{G}\eta(T) \tag{2}$$

where g_{l} is the liquid volume fraction and u_{l} is the liquid velocity in the mushy zone. The function, $\eta(T)$, is the integrated solid fraction across the mushy zone expressed as a function of temperature instead of distance by change of variables as

$$\eta(T) = \int_{T_{\mathrm{sol}}}^{T} (1 - g_{\mathrm{l}})\mathrm{d}T \tag{3}$$

where T_{sol} is the temperature at which the alloy is fully solidified. Equation 2 is exactly the same expression found by Rappaz et al.[10] It is interesting to note that with this arrangement of terms, the velocity in the liquid is driven by the cooling rate and the strain rate multiplied by some factor of the liquid fraction and density. The pressure drop in the

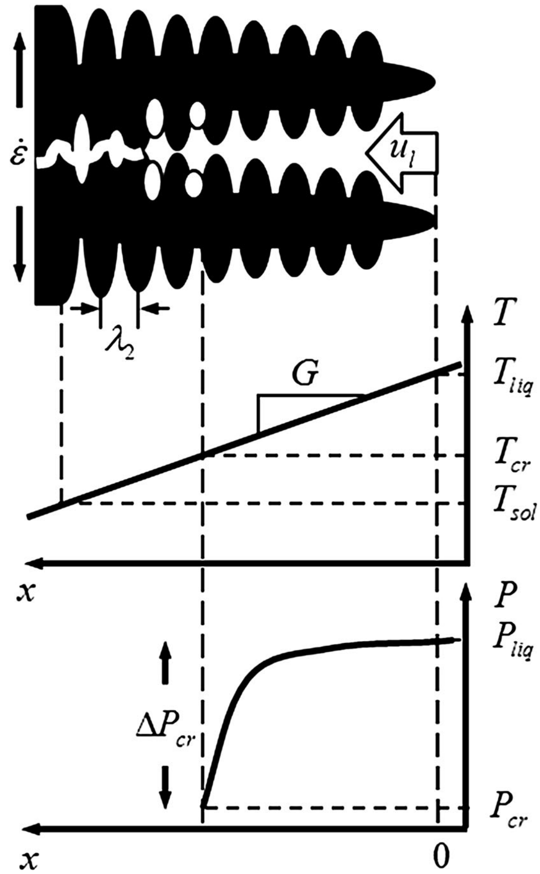

Fig. 1. Schematic of the one-dimensional mushy zone.

mushy zone can be found using Darcy's law written here as

$$g_l u_l = -\frac{K}{\mu_l}\frac{dP}{dx} \tag{4}$$

where μ_l is the liquid dynamic viscosity and P is the melt pressure. The permeability, K, in the mushy zone can be found using the Kozeny–Carman relation and can be expressed for this situation as

$$K = \frac{\lambda_2^2}{180}\frac{g_l^3}{(1-g_l)^2} \tag{5}$$

where λ_2 is the secondary arm spacing (SDAS). Combining Eqs. 2 and 4 results in

$$\frac{dP}{dx} = \frac{\mu_l \beta \dot{T} g_l}{KG} + \frac{\mu_l(1+\beta)\dot{\varepsilon}\eta(T)}{KG} \tag{6}$$

Figure 1 illustrates that as the liquid fraction decreases, the pressure drops from the liquid pressure to a lower pressure in the mushy zone. The initial pressure in the liquid depends on the macroscopic solidification situation. Early in solidification, the liquid pressure, P_{liq}, is the metallostatic pressure within the casting. The pressure will drop in the mushy zone to some critical pressure, P_{cr}, yielding a critical pressure drop $\Delta P_{cr} = P_{liq} - P_{cr}$. This critical pressure drop will be a function of the critical nucleation radius of the porosity and the pore gas pressure from the Young–Laplace equation. The uncertainty in the critical pressure at which pores nucleate is large, and therefore, the critical pressure drop ΔP_{cr} is simply treated as an adjustable parameter of the model. Equation 6 can be integrated over the mushy zone from the critical point to the liquidus isotherm. Using a change of variables to express this integration as a function of the liquid fraction, the following expression can be obtained as

$$\Delta P_{cr} = \mu_l \beta \frac{\dot{T}}{G^2}\int_{g_{l,cr}}^{1}\frac{g_l}{K}\frac{dT}{dg_l}dg_l + \mu_l(1+\beta)\frac{\dot{\varepsilon}}{G^2}\int_{g_{l,cr}}^{1}\frac{\eta(T)}{K}\frac{dT}{dg_l}dg_l \tag{7}$$

where $g_{l,cr}$ is the critical liquid fraction at which the melt pressure drops to the critical pressure when porosity begins to form. Equation 7 is the same expression as the one found by Carlson and Beckermann with the addition of a second term that accelerates the pressure drop due to deformation. This expression is also identical to the one found by Rappaz et al.; however, the current expression depends on integration from the temperature at the critical liquid fraction rather than on integration over the entire temperature range and then determining the maximum strain rate.

Equation 7 can be made dimensionless by introducing a new, scaled temperature, $\theta = (T - T_{sol})/\Delta T_f$, where $\Delta T_f = T_{liq} - T_{sol}$ is the freezing range and T_{liq} is the liquidus temperature. Using this definition along with Eq. 5 yields

$$\Delta P_{cr} = \frac{\mu_l \beta \Delta T_f}{\lambda_2^2}\frac{\dot{T}}{G^2}I_{sh}(g_{l,cr}) + \frac{\mu_l(1+\beta)\Delta T_f^2}{\lambda_2^2}\frac{\dot{\varepsilon}}{G^2}I_d(g_{l,cr}) \tag{8}$$

where

$$I_{sh}(g_{l,cr}) = \int_{g_{l,cr}}^{1} 180\frac{(1-g_l)^2}{g_l^2}\frac{d\theta}{dg_l}dg_l \tag{9}$$

and

$$I_d(g_{l,cr}) = \int_{g_{l,cr}}^{1} 180\frac{\eta^*(g_l)(1-g_l)^2}{g_l^3}\frac{d\theta}{dg_l}dg_l \tag{10}$$

The dimensionless version of $\eta(T)$ is

$$\eta^*(g_{l,cr}) = \int_{0}^{g_{l,cr}} (1-g_l)\frac{d\theta}{dg_l}dg_l \tag{11}$$

These integrals can be found either analytically or numerically using available alloy solid fraction-temperature curve data. This feature is particularly useful when considering industrially relevant multicomponent alloys. The first integral, $I_{\rm sh}(g_{\rm l,cr})$, is associated with the term related to shrinkage, and the second integral, $I_{\rm d}(g_{\rm l,cr})$, is associated with the term related to the deformation. Note that using Eq. 8 is the same as the RDG criterion, if the integration is performed over the entire solidification range and solved for the strain rate that satisfies the critical pressure drop. Herein lies the improvement beyond the RDG criterion; the imposed strain causes the critical liquid fraction to be lower than due to shrinkage alone.

Using the Niyama criterion given in Eq. 1 and rearranging Eq. 8, a dimensionless Niyama criterion, Ny*, can be defined as

$$\mathrm{Ny}^* = \sqrt{\frac{\Delta P_{\rm cr}\lambda_2^2}{\beta\mu_{\rm l}\Delta T_{\rm f}}}\frac{G}{\sqrt{\dot{T}}} \tag{12}$$

and a new integrated characteristic strain, ε^*, can be defined as

$$\varepsilon^* = \frac{\Delta T_{\rm f}\dot{\varepsilon}}{\dot{T}} \tag{13}$$

These definitions allow us to simplify Eq. 8 as

$$\mathrm{Ny}^* = \sqrt{I_{\rm sh}(g_{\rm l,cr}) + \frac{1+\beta}{\beta}\varepsilon^* I_{\rm d}(g_{\rm l,cr})} \tag{14}$$

This expression for the dimensionless Niyama is identical to the previously developed expression[11] with one additional term to adjust the value under the application of an applied strain. The dimensionless Niyama criterion can be calculated from casting simulation software in the same way it currently is calculated. The characteristic strain can either be specified directly or otherwise obtained from a constant strain rate that is either estimated or directly calculated in a casting stress simulation. It should be evaluated at the same temperature at which Ny is calculated and using a strain rate characteristic of deformation normal to the thermal gradient. In three dimensions, this may be chosen as the volumetric or deviatoric strain rate. With this information, Ny* and ε^*, the critical liquid fraction, $g_{\rm l,cr}$, can be determined. This procedure is the same as for the dimensionless Niyama without applied deformation. When flow stops at the critical liquid fraction, mass conservation can be integrated and simplified to find the porosity that must exist to feed the remaining shrinkage and deformation. The shrinkage related porosity is given as

$$g_{\rm p,sh} = \frac{\beta}{1+\beta}g_{\rm l,cr} \tag{15}$$

and the deformation related porosity can be found using the same scaling variables from before as

$$g_{\rm p,d} = \varepsilon^*\eta^*(g_{\rm l,cr}) \tag{16}$$

The use of Eqs. 15 and 16 is a novel extension of the utility of the dimensionless Niyama approach greatly enhancing the utility of this criterion for the prediction of hot tearing. This approach yields a simple consistent way to explore both the shrinkage porosity as well as the hot tearing severity within the same method. Using results from casting simulation software for the Niyama criterion and the strain rate, both porosity due to shrinkage and deformation can be directly calculated.

SHRINKAGE AND DEFORMATION POROSITY PREDICTION

The dimensionless Niyama criterion developed in the previous section is now applied to a binary aluminum alloy over a range of copper concentrations. As in the RDG paper, the solid fraction-temperature curve is calculated using the following expression from Kurz and Fisher:[13]

$$g_{\rm s}(T) = 1 - g_{\rm l} = \frac{1}{1-2\alpha_{\rm s}^* k}\left[1 - \left[\frac{T_{\rm m}-T}{T_{\rm m}-T_{\rm liq}}\right]^{\frac{1-2\alpha_{\rm s}^* k}{k-1}}\right] \tag{17}$$

where k is the partition coefficient, $T_{\rm m}$ is the melting point of pure aluminum, and $\alpha_{\rm s}^*$ is a back diffusion coefficient. The liquidus temperature is given by

$$T_{\rm liq} = T_{\rm m} - m_{\rm liq}C_0 \tag{18}$$

where $m_{\rm liq}$ is the liquidus line slope and C_0 is the solute concentration of the alloy. Table 1 provides the parameters used to illustrate the quantities developed in the "Method" section.

The solidification conditions, such as the thermal gradient and the cooling rate, were chosen to be typical for shape casting. For the example shown in this work, a constant critical pressure drop of the melt pressure equal to atmospheric pressure or 101 kPa was used. This critical pressure drop is significantly higher than the value chosen in the RDG criterion study at 2 kPa. Higher values of critical pressure drop lead to lower values of porosity.

Figure 2 shows the various integrals as a function of the critical liquid fraction for an Al-3 wt.% Cu alloy. All of the integrals have a one-to-one correspondence to the critical liquid fraction. For this composition, the volume fraction of eutectic is about 5%, as indicated in the figure by a dashed line. If the critical liquid fraction found from the dimensionless Niyama criterion is less than this value, the eutectic

Table I. List of parameters used in the calculation of the dimensionless Niyama criterion and shrinkage porosity calculations

Parameter	Symbol	Value	Unit
Viscosity	μ_l	1.0	mPa s
Critical pressure drop	ΔP_{cr}	101	kPa
Total solidification shrinkage	β	0.06	–
Thermal gradient	G	200	K/m
Cooling rate	$\dot{T}$	4.0	K/s
Secondary arm spacing	λ_2	0.0001	M
Partition coefficient	k	0.155	–
Back diffusion coefficient	α_s^*	0.01	–
Liquidus line slope	m_{liq}	3.37	K/wt.%
Melting temperature of pure metal	T_m	933	K
Eutectic temperature	T_e	855	K
Alloy concentration	C_0	3.0	wt.%
Strain rate	$\dot{\varepsilon}$	0.001	1/s

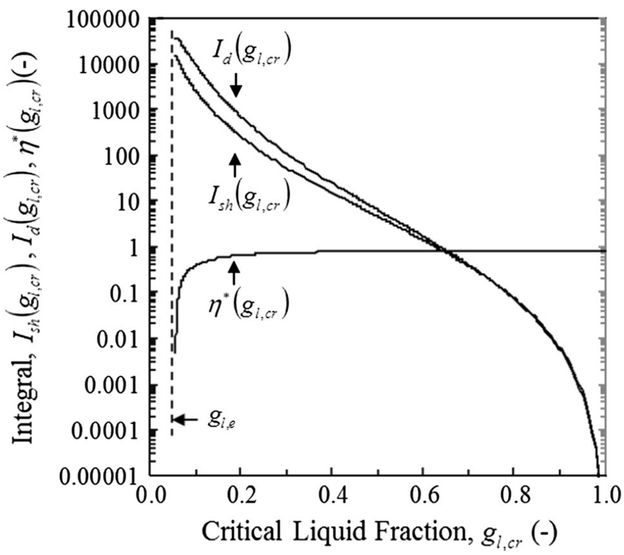

Fig. 2. Evolution of the integrals from the derivation with respect to the critical liquid fraction.

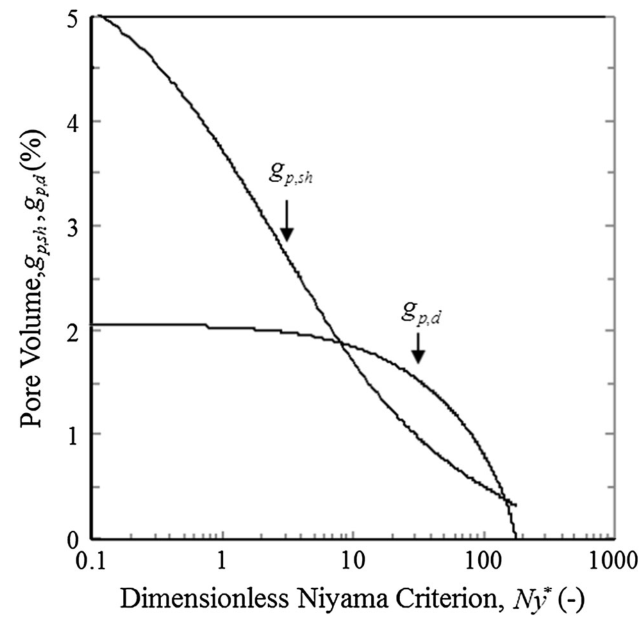

Fig. 3. Predicted shrinkage and deformation pore volume as a function of the dimensionless Niyama criterion for a concentration of 3 wt.% Cu with an applied strain rate of 10^{-3} (1/s).

formation can be expected to prevent any shrinkage or deformation porosity.

Using the results in Fig. 2 together with the parameters given in Table I, the critical liquid fraction value can now be calculated from Eq. 8. Then, the values for the shrinkage and deformation porosity can be found from Eqs. 15 and 16, respectively. Figure 3 shows the predicted shrinkage and deformation pore volume fractions as a function of the dimensionless Niyama criterion, Ny^*. The maximum shrinkage porosity fraction is given by the total solidification shrinkage. The maximum deformation porosity fraction depends on the maximum value of the η^* integral and the characteristic strain. Lower values of the dimensionless Niyama criterion yield higher levels of both shrinkage and deformation porosity, but the deformation porosity becomes almost independent of the Niyama criterion for $Ny^* < 10$. Note that at higher values of the dimensionless Niyama criterion, the deformation porosity becomes greater than the shrinkage porosity. However, the magnitude of the deformation porosity depends strongly on the strain rate, and for lower values of the strain rate (10^{-3} 1/s in Fig. 3), the deformation porosity may not exceed the shrinkage porosity volume fraction. In the present alloy, the presence of eutectic leads to an upper limit for the dimensionless Niyama criterion above which no porosity is predicted to form. Criterion values above this limit are possible, but they do not lead to the formation of porosity. This restriction is expected because for sufficiently high values of the Niyama criterion, the pressure drop in the mushy zone never exceeds the critical pressure drop.

Figure 4 shows the effect of an increasing strain rate on the shrinkage and deformation porosity fraction for a constant value of 100 ((K s)$^{0.5}$/m) for the Niyama criterion. The deformation porosity increases almost linearly with the strain rate. On the other hand, the shrinkage porosity increases only slightly with strain rate, which can be attributed to the fact that the critical liquid fraction is only a weak function of the strain rate. The deformation porosity becomes less than the shrinkage porosity for strain rates below about 6×10^{-4} (1/s). Even for relatively low values of the deformation porosity, deformation is expected to align the porosity perpendicular to the tensile direction, which then gives the typical appearance of a hot tear.

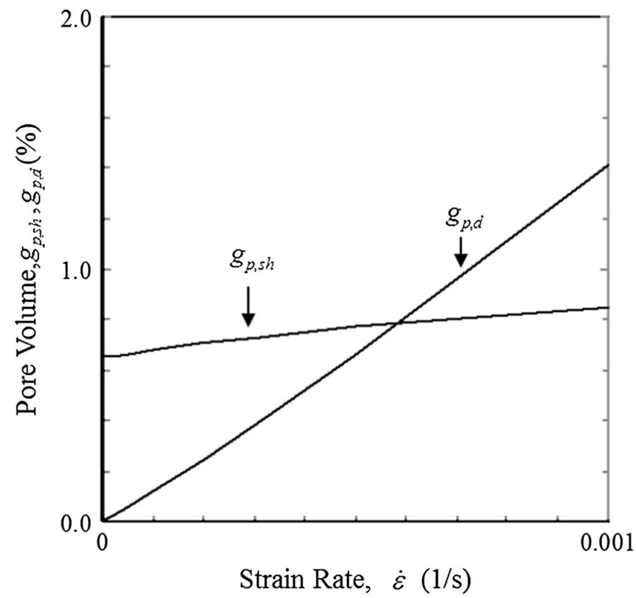

Fig. 4. Predicted shrinkage and deformation pore volume as a function of the strain rate for a concentration of 3 wt.% Cu with a Niyama criterion value of 100 ((K s)$^{0.5}$/m).

HOT TEAR Λ-SHAPED CURVE PREDICTION

Using the solid fraction-temperature curve given by Eq. 17 and the parameters provided in Table I, the effect of alloy composition on hot tearing is investigated. Figure 5 shows the predicted shrinkage and deformation porosity volume fractions as a function of the copper content for two values of the Niyama criterion. Typical hot tear Λ-shaped curves are obtained. The maximum hot tearing potential occurs at a concentration of 0.248 wt.% Cu as measured by the magnitude of the deformation porosity. This concentration, $C_{0,\max}$, corresponds to the maximum solidification temperature interval. The maximum solidification interval occurs when the copper content is sufficiently large that some eutectic forms. Taking $T = T_e$, where T_e is the eutectic temperature, and solving Eq. 17 for concentration, gives

$$C_{0,\max} = \frac{T_m - T_e}{m_{liq}\left(1 - \left(1 - g_{l,b}\right)\left(1 - 2\alpha_s^* k\right)\right)^{\frac{k-1}{1-2\alpha_s^* k}}} \quad (19)$$

where $g_{l,b}$ is the liquid fraction at which interdendritic bridging occurs.[10] For the parameters given in Table I, the maximum solidification interval concentration is 0.248 wt.% Cu if $g_{l,b}$ is taken to be equal to zero. In the RDG paper,[10] the maximum hot tearing potential was found to occur at a concentration of 1.36 wt.% Cu. This concentration can be obtained from Eq. 19 by setting the liquid fraction for bridging, $g_{l,b}$, equal to 0.02.

In Fig. 5, finite deformation and shrinkage porosity values are found up to a concentration of 7 wt.% Cu for a Niyama criterion value of 100 ((K s)$^{0.5}$/m) and up to a concentration of 3 wt.% Cu for a Niyama criterion value of 400 ((K s)$^{0.5}$/m). At copper concentrations above this limit for a given Niyama criterion value, eutectic forms before the critical liquid fraction is reached, which prevents any porosity from nucleating. The results in Fig. 5 show that this limit in the concentration is not only a function of the alloy properties, but also of the thermal conditions (temperature gradient and

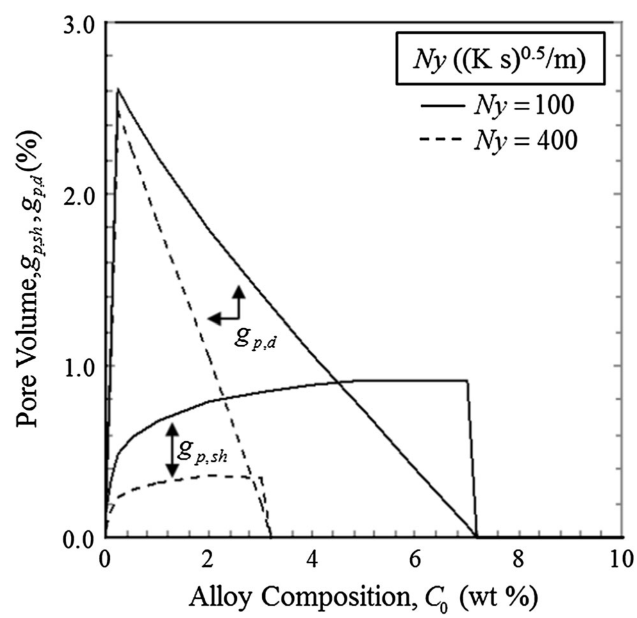

Fig. 5. Predicted shrinkage and deformation pore volume as a function of alloy concentration for a strain rate of 10^{-3} (1/s) with two values of Niyama criterion, 100 ((K s)$^{0.5}$/m) and 400 ((K s)$^{0.5}$/m).

cooling rate) during casting as expressed by the Niyama criterion. Although the thermal conditions are not well characterized in the hot tear experiments of Spittle and Cushway,[1] the Λ-shaped curve for a Niyama criterion value of 400 ((K s)$^{0.5}$/m) corresponds most closely to the experimental data, as plotted in Fig. 6 of the RDG paper.[10]

While the deformation porosity has the Λ-shaped curve behavior shown in Fig. 5, the shrinkage porosity increases monotonically with copper content from zero until eutectic forms before the critical liquid fraction is reached. As expected, the shrinkage porosity percentages are higher for the lower Niyama criterion value.

CONCLUSION

A dimensionless form of the Niyama criterion accounting for both shrinkage and deformation porosity is presented. Both shrinkage and deformation lead to a change in the liquid velocity and therefore to the pressure drop in the mushy zone. A critical liquid fraction when the liquid velocity stops can be determined. Solidification of the remaining liquid leads to shrinkage porosity. Deformation porosity can be found from the applied strain and acts to change the amount of shrinkage porosity. The dimensionless Niyama criterion provides the magnitude of the shrinkage porosity as well as the severity of hot tearing through the deformation porosity volume fraction. It is shown that the deformation porosity varies with strain rate and alloy composition as expected from hot tearing experiments. The present dimensionless Niyama criterion can be easily implemented in casting simulation software to predict both shrinkage porosity and hot tearing susceptibility.

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