Reliability based casting process design optimisation

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Deterministic optimum designs are unreliable without consideration of the statistical and physical uncertainties in the casting process. In the present research, casting simulation is integrated with a general purpose reliability based design optimisation (RBDO) software tool that considers uncertainties in both the input variables as well as in the model itself. The output consists of an optimal design that meets a specified reliability. An example casting process design is presented where the shape of a riser is optimised while considering uncertainties in the fill level and riser diameter. It is shown that RBDO provides a much different optimum design than a traditional deterministic approach. The deterministic optimal solution offers a 12% increase in casting yield over typical safety margin design practice, but has an unacceptable 61% probability of failure. The RBDO design has a 7% increase in casting yield over the safety margin approach and a probability of failure of 4·6%.

Keywords: Casting process design, Optimisation, Probability of failure, Reliability based design optimisation

Introduction

Casting process simulation has become an invaluable tool in the production of economical and high performance cast components. Its application by experienced and knowledgeable operators leads to reduced casting defects, casting yield improvement and reduced trial and error iteration in development of a casting's rigging. Increasingly, casting simulation is being used as a collaborative tool between component designers and casting producers to reduce lead times, to develop casting friendly component designs and to produce better castings. The majority of casting simulation is being used in a purely deterministic approach, replacing iterative trial and error process development on the shop floor with iterations on the computer. In this purely deterministic approach, the experience and knowledge of the engineer operating the software determines to a great extent that the software is used effectively and that the casting process developed is the best it can be.

To maximise the effectiveness of casting simulation and improve the likelihood of an operator achieving an optimal solution, automatic optimisation algorithms for casting process development are being researched,[1–5] and commercial software such as MAGMAfrontier,[6–9] OPTICast[10] and AutoCAST-X[4,11,12] have been developed. The most common application found in all of these automatic process optimisation tools is the solution to the problem of casting feeding system optimisation. In optimisation of the casting feeding system, optimal sizes and locations of feeders are determined such that casting yield (ratio of mass of casting produced to mass of metal poured) is maximised and the desired quality level is met, which is typically defined as an absence of, or low level of, shrinkage porosity in the casting. Despite the power and promise of these developments in casting process optimisation, there are major shortcomings in these purely deterministic optimisation approaches: neither the reliabilities of the casting production process nor the reliabilities of the casting model are considered. Uncertainties in the casting process conditions and variables, and in the casting model parameters and properties must be considered since these will affect the feasibility (i.e. reliability) of the optimised solution. Casting process optimisation where the feasibility of the process solution is not considered is termed deterministic design optimisation (DDO). In DDO, the probability of success or failure of the solution is not calculated. Determination of the optimum solution's probability of success is based largely on the software operator. Based on their experience, they must judge the feasibility of the solution and make adjustments if needed. Reliability based design optimisation (RBDO), on the other hand, considers uncertainties in the variables of the design problem in determining an optimum design solution that meets a target probability of success. In RBDO, even the uncertainties of the models used in solving the optimisation problem can be considered to determine a fully reliable optimum solution.

Here, the authors present an optimisation study for a casting process design using the recently developed, general purpose, Iowa Reliability-Based Design Optimisation (I-RBDO) software.[13–16] Using uncertainties in the design process variables, and uncertainties in analytical models and variables used in design processes, the I-RBDO tool determines optimum designs that meet target

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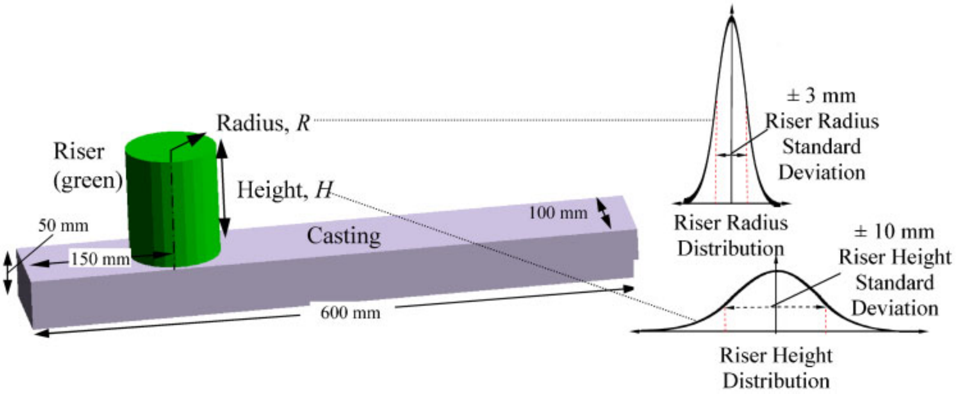


1 Casting (shown in light purple) with dimensions and riser (shown in green) used in casting feeding system design case study. Distributions for riser radius and height assumed for RBDO analysis are also shown

reliability levels satisfying performance measures and constraints that designers specify. The I-RBDO method has been applied to a range of design problems such as structural design,[13,16] fluid–solid interations,[17] reducing residual deformation in welding,[18] magnetic energy storage systems[19] and multibody system dynamics.[20] The objective of the current study is to investigate use of I-RBDO with casting simulation.

The casting process design optimisation methods presented here are demonstrated using the steel casting shown in Fig. 1. The casting consists of a rectangular bar that is fed by a cylindrical top riser. The goal is to use simulation predictions to produce a steel casting that is free of shrinkage porosity. The design of the casting feeding system will be performed using a typical industrial approach where a safety margin between the casting surface and the bottom of the riser shrinkage piping is used to determine the riser size. The safety margin approach will then be compared to optimised risers designed using DDO and RBDO methods, where the optimum design uses the smallest riser volume giving a porosity free casting. The design variables are the riser diameter and height, and for the RBDO analysis, the statistical uncertainties in these variables are represented by the probability distributions shown schematically in Fig. 1. By considering these uncertainties in the casting production process design, RBDO provides an optimal casting process design that meets the target reliability level within the porosity free constraint that the casting process designer specifies.

To the authors' knowledge, this is the first application of the RBDO method to casting process design. As a result, the intent of this paper is to demonstrate the concepts behind the RBDO method rather than all technical details, which are given in the references cited here. The optimisation problem considered here is relatively simple, but it is well defined. This problem serves well to demonstrate the RBDO technique, allowing it to be readily visualised and compared with the DDO method used. This study, as do most design optimisation studies, also provides insight into the design and the effects of variables on the design's objectives, even for a seemingly simple optimisation problem as this. Tackling a more complex casting process design optimisation problem with numerous variables and constraints is outside the scope of this paper, which is to introduce the RBDO method to the casting community.

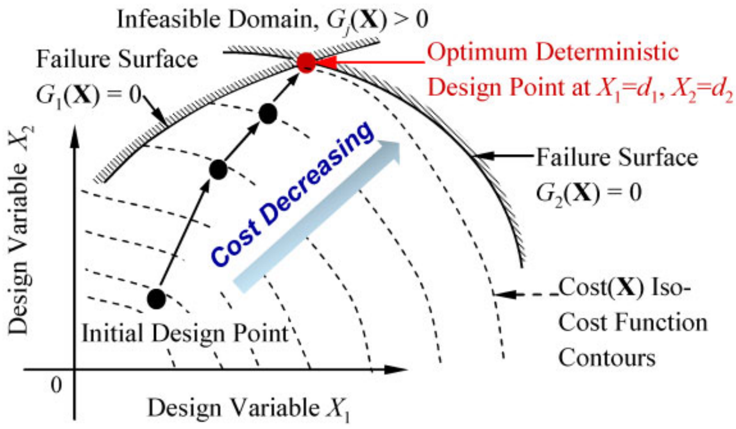
Overview of DDO and RBDO optimisation methods

Whether by DDO or RBDO methods, casting process design optimisation inherently involves multiple variables, multiobjectives and multiconstraints. Many of the objectives conflict, such as achieving a porosity free casting with the smallest size feeders. As such, the most successful optimisation algorithms developed for casting process optimisation with conflicting objects have been the multiobjective evolutionary algorithms[2] or multiobjective genetic algorithms such as modeFRONTIER,[21] which has been implemented in the casting process optimisation software MAGMAfrontier.[6–9] As has been demonstrated,[7] given casting process and model variables (i.e. initial rigging, metal and mould properties, heat transfer coefficients, pouring temperature and time, etc.), and given process constraints (i.e. porosity level, other defects, customer requirements, alloy, etc.), an optimised process can be determined to meet required objectives (i.e. maximum casting yield, required mechanical properties, etc.). These applications of optimisation algorithms to casting processes are deterministic as described below. In the optimisation problem solved in the current work, there is a single objective function to minimise the volume of the riser. The present version of the I-RBDO software addresses problems having a single objective function with multiple variables and constraints as described below.

A general DDO problem is commonly defined in terms of a single objective 'Cost' function of design variables to be minimised as

$$\begin{aligned}&\text{Minimise Cost}(\mathbf{X})\\&\text{subject to constraint or performance}\\&\text{functions } G_j(\mathbf{X})\le 0 \text{ for } j=1,\ldots,nc\end{aligned}\tag{1}$$

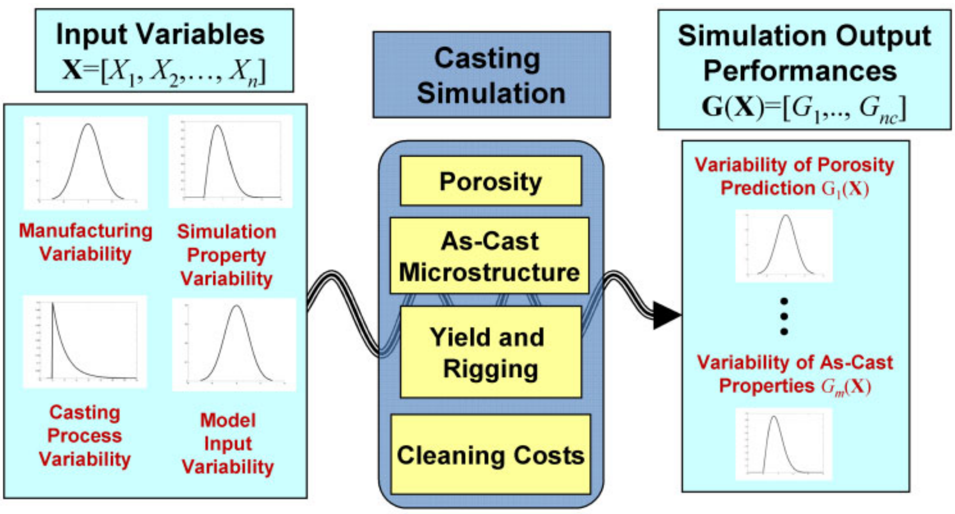
where $\mathbf{X}=\{X_1, X_2, \ldots, X_{nd}\}^T$ is the vector of nd input design variables, and nc is the number of constraint functions defined such that the jth constraint is violated when $G_j(\mathbf{X})>0$. Typically, there is a design space defined for the vector of input design variables $\mathbf{X}$ such that they must fall between upper and lower bounds, $\mathbf{X}^U$ and $\mathbf{X}^L$ respectively, so that $\mathbf{X}^L\le\mathbf{X}\le\mathbf{X}^U$. In equation (1), it is important to point out that the input design variable vector is deterministic. In the casting process DDO problem studied here, the cost function Cost($\mathbf{X}$) is the total amount of metal poured to produce the casting,



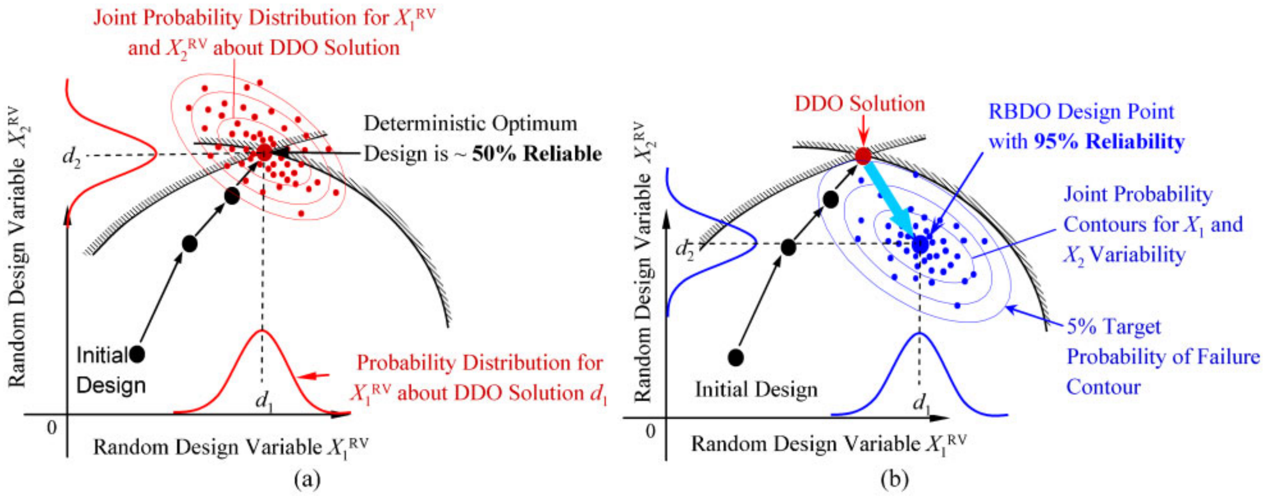
2 Visualised solution process to general DDO problem for two input design variables (X_1, X_2) and two performance constraint functions (G_1, G_2)

and this cost is minimised when the casting yield is maximised. The solution process to the general DDO problem can be visualised for two input design variables (X_1, X_2) and two performance constraint functions (G_1, G_2) as is shown in Fig. 2. Here, the cost function Cost($\mathbf{X}$) is shown by the iso-Cost function value contour lines indicated by dashed lines, where the direction of decreasing cost is shown. The two performance constraint functions are visualised in Fig. 2 by showing the boundary curves between acceptable values of $G_j(\mathbf{X})\leq 0$ and the infeasible domain on the hatched side of the curves where $G_j(\mathbf{X})>0$. These boundaries, where $G_1(\mathbf{X})$ and $G_2(\mathbf{X})$ are 0, are termed failure surfaces. Depending on the optimisation algorithm used,[2,6–9,21] the solution process begins at the initial design point as shown in Fig. 2 and advances to the optimum design point at $X_1=d_1$ and $X_2=d_2$, such that equation (1) is satisfied. This optimum is shown by the red design point in Fig. 2. Though visualised in Fig. 2 as well defined contours and smooth functions, in the present work, they are not necessarily so, since the cost function and performance constraints are results from casting simulation software. The I-RBDO software tool used here can use the results from any simulation package, or combination of packages, to provide the cost function and performance measures in a completely general way.[13,16–20]

For reliability based optimisation, both the multiobjective aspect of the problem through the multiple constraints $G_j(\mathbf{X})$ and the uncertainties in the casting manufacturing process and model input variables must be defined. In Fig. 3, the input variables and output performance measures and constraints, and their uncertainty distributions, are shown for a casting process and its simulation. For example, the variability of model inputs such as computational grid used, filling conditions, model algorithms and accuracy, actual casting process variability, cast metal and mould thermophysical simulation properties used and manufacturing variability of mould cavity dimensions might be considered in reliability based optimisation. In the optimisation problem considered here, two input process design variables and their uncertainties are used in the RBDO analysis, the riser height and diameter dimensions shown in Fig. 1. These variables are dependent on the casting process. As indicated in Fig. 3 under the 'Casting Simulation' block, there are important predictions from the casting simulation that can affect the performance of the casting process design, such as defect predictions like porosity and inclusions or resulting structural material properties of the casting arising from its as cast microstructure. Here, porosity prediction is used as a performance constraint, but in a more complex



3 Diagram of input variables, output performances measures and constraints, and their uncertainty distributions, for casting process and its simulation



4 Visualisation of RBDO problem dependent on two random variables X_1^{RV} and X_2^{RV} showing *a* that DDO solution is only about 50% reliable at design points d_1 and d_2 when their variability is considered, and *b* solution process to equation (2) visualised where design point is shifted to minimum cost function value at new design points such that target probability of failure is satisfied for joint probability distribution and failure surfaces

optimisation problem, other possible variables shown in Fig. 3 might also be considered. In the I-RBDO software used in the current work, uncertainties and variations in the casting process variables and the casting modelling software variables and parameters are described via statistical distributions. Normal, Lognormal, Weibull, Gamma, Gumbel and Extreme I and Extreme II distributions may be used.

The differences between the RBDO and DDO solutions are presented in Fig. 4. Again using the two design variable optimisation problem visualised in Fig. 2, when the variabilities of X_1 and X_2 are considered, probability distributions exist about each of the design points d_1 and d_2 at the DDO solution. This is the starting point of the RBDO problem formulation, where the deterministic design variable vector **X** from equation (1) is replaced by a random design variable vector $\mathbf{X}^{RV}=\{X_1^{RV}, X_2^{RV}, \ldots, X_{nd}^{RV}\}^T$. In Fig. 4*a*, two random variables and their distributions X_1^{RV} and X_2^{RV} are shown on the abscissa and ordinate axes with the means of these distributions at design points d_1 and d_2. In both subfigures of Fig. 4, the cost function contours and failure surfaces are unchanged from Fig. 2. Showing them again is unnecessary. In the general RBDO problem formulation, there is a variable design vector $\mathbf{d}=\{d_1, d_2, \ldots, d_{nd}\}^T$, which is made up of mean values of each of the *nd* random design variables. The mean value operator is denoted by $\boldsymbol{\mu}(\cdot)$, so that $\mathbf{d}=\boldsymbol{\mu}(\mathbf{X}^{RV})$. In this two-dimensional design variable example, a joint probability distribution for the random design variables X_1^{RV} and X_2^{RV} is formed by their product about the DDO solution as seen in Fig. 4*a*. Here, the joint probability distribution is represented by the three red ellipse shaped contour lines about the DDO solution. In general, for any number of independent random design variables *nd*, the joint probability distribution is formed by the product of all the marginal probability distributions of the design variables.[16] As can be readily seen in Fig. 4*a*, and visualised by the series of red points representing random samples about the DDO solution, roughly 50% of the distribution and random samples fall outside the failure surfaces and in the infeasible domain. The deterministic optimum design is only about 50% reliable, which is often the case with DDO solutions. The RBDO solution addresses this shortcoming of the DDO solution.

Using the concepts visualised in the discussion of Fig. 4*a*, a general RBDO problem is defined in terms of a cost function of the design vector **d** to be minimised as

$$\begin{aligned}&\text{Minimise Cost}(\mathbf{d})\\&\text{Subject to } P_{\mathrm{Fj}}(\mathbf{d})=P[G_j(\mathbf{X}^{RV})>0]\leq P_{\mathrm{Fj}}^{\mathrm{Tar}}\\&\text{for } j=1,\ldots,nc \text{ and } \mathbf{d}^{L}\leq\mathbf{d}\leq\mathbf{d}^{U}\end{aligned}\tag{2}$$

where $P_{\mathrm{Fj}}(\mathbf{d})$ is defined as the probability of failure for the *j*th constraint function at the design vector **d**, G_j is the *j*th constraint function, $P[\cdot]$ is the probability measure such that $P[G_j(\mathbf{X}^{RV})>0]$ is the probability that the *j*th constraint function is violated, *nc* is the number of constraint functions, $P_{\mathrm{Fj}}^{\mathrm{Tar}}$ is the target probability of failure for the *j*th constraint function, and $\mathbf{d}^L$ and $\mathbf{d}^U$ are the lower and upper bounds respectively, defining the design space. The solution process to equation (2) is visualised for two random design variables in Fig. 4*b*, where the design point must be shifted to the minimum cost function value such that the target probability of failure is satisfied for the joint probability distribution relative to the failure surfaces. For example in Fig. 4*b*, if the target probability of failure for both constraints is 5%, and if only 5% of the joint probability distribution falls outside the outermost joint probability contour drawn in Fig. 4*b*, then that outermost probability contour corresponds to the target probability of failure contour of 5%. Equation (2) is satisfied when the outermost probability contour drawn in Fig. 4*b* falls completely inside the feasible design domain [where $G_j(\mathbf{X}^{RV})\leq 0$] such that Cost(**d**) is minimised. For example, with a 5% target probability of failure, the design point shifts from the DDO solution, which is only ~50% reliable in Fig. 4*a*, to an RBDO design point, which is 95% reliable in Fig. 4*b*. In general, the target probabilities of failure for the *nc* constraints in equation (2) need not be equal as in this visualised example, which would alter the shifting in the design point relative to a given constraint's failure surface.

Sensitivity and sampling based optimisation and I-RBDO method

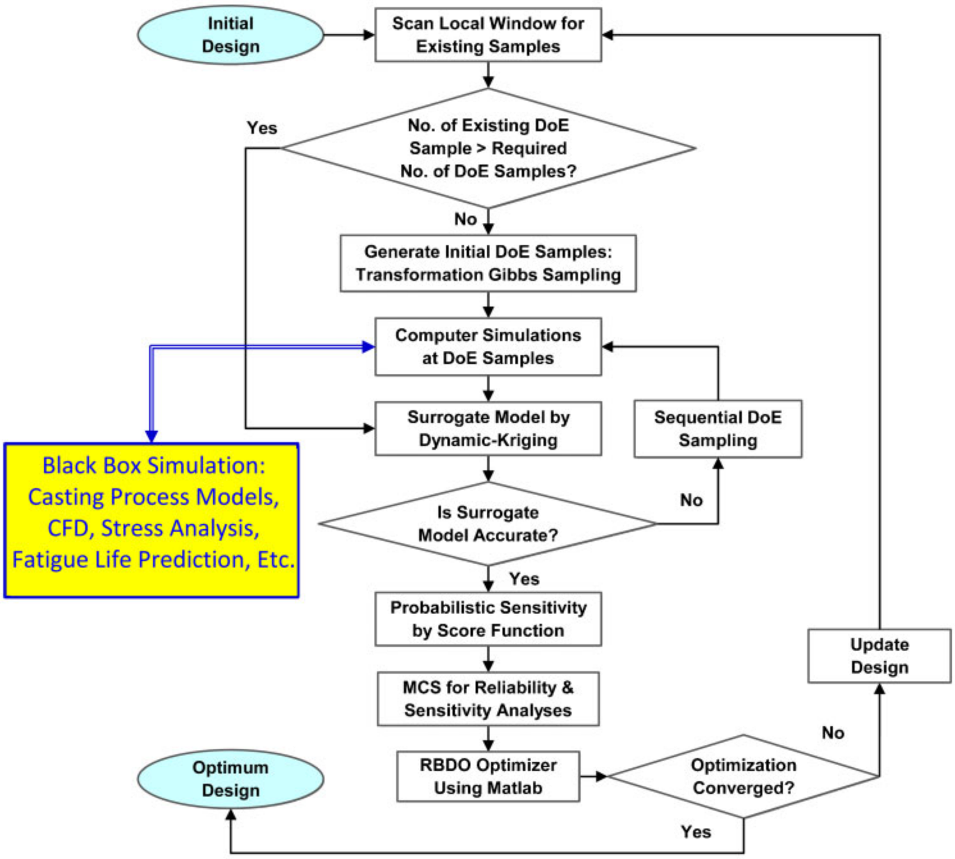
The details of the reliability based optimisation algorithms used in the I-RBDO software is beyond the scope of the current paper and have been presented in detail elsewhere.[13–16,22] This section is provided to give an overview of the fundamental concepts and terminology related to RBDO. In most conventional design optimisation algorithms, evaluations of the cost and objective functions in equations (1) and (2) are performed using 'true' evaluation of the functions. In the current application example, such 'true' function evaluations are the results of porosity predictions from computer modelling of the casting process and the riser volume calculation. Typically in textbook optimisation algorithms,[23,24] the sensitivities of the 'true' function evaluations to changes in the design variables are used in the solution algorithm to guide the solution to the minimum cost satisfying the constraints for the problem through a series of steps of function evaluations. This is termed design sensitivity analysis. This is visualised for example in Fig. 2 for the series of design points marching towards the optimum point, where the direction of the decreasing cost function satisfying the constraints could be determined from its gradient with respect to the two design variables. An excellent example of applying such a design sensitivity analysis technique to optimal riser design for a porosity free casting was referenced earlier,[1] where 'true' function evaluations were made using two-dimensional casting simulations to predict porosity. The drawback of sensitivity based optimisation is that, while the sensitivity based RBDO method is very effective and robust, there are engineering design problems, such as casting process design, for which design sensitivity information cannot be obtained readily or is computationally expensive. For these design problems, an alternative method needs to be developed. For such optimisation problems, sampling based optimisation methods have been developed, which replace evaluations of the 'true' function with results from a much less computationally expensive surrogate model.[13] Surrogate models (also termed metamodels) are developed from a limited number of evaluations of the 'true' function evaluations. Furthermore, use of surrogate models becomes quite advantageous in RBDO calculations when computing the probability of failure of a design. The probability of failure calculations performed here is carried out using 200 000 Monte Carlo points that are evaluated using the surrogate model. This is performed for every iteration of optimisation. Performing 200 000 casting simulations for each optimisation iteration would be impractical. Another advantage of sampling based optimisation is that the fidelity of the surrogate model to the true model is known by determining the error between surrogate model evaluations and the true function evaluations that are used to develop it. The surrogate model accuracy can be tailored to the desired accuracy while considering the trade off in the computation expense of true function evaluations required to create it.

In the current work, both the DDO and the RBDO calculations use a sequential quadratic programming optimisation algorithm,[23,24] where the casting simulation model is approximated using dynamic Kriging (DKG) models.[13,14] The DKG method fits the true model more accurately than the universal Kriging method by improving four areas: (i) using genetic algorithm and generalised pattern search algorithm for parameter search in the maximum likelihood estimate, (ii) using the penalised maximum likelihood estimate for small design of experiments (DoEs) sample size, (iii) correlation model selection using maximum likelihood estimate, and (iv) selection of the mean structure using the cross-validation error. For sampling based RBDO, stochastic design sensitivity analysis is carried out using the score function.[15,16]

The computational flow diagram of the sampling based RBDO used in the current work is shown in Fig. 5. When the RBDO algorithm starts at the initial design, it determines the minimum number of DoE, i.e. the minimum number of casting simulations, required to create the DKG models. It checks to see if any previously run casting simulations are within the local window for the current design. The local window is a subdomain of the entire design space of the design variables. A local window is used because it is more efficient to create the DKG models within a given local window than it is to use the entire design space.[16] Details can be found in the reference. The local window is centred around the current design point, and the size of the local window is determined from the input distributions. If no DoE samples are found or if there are not enough samples to satisfy that the DKG model is sufficiently accurate, the code generates random uniform initial DoE samples within the local window. The casting simulations for these DoEs are then carried out. Using the casting simulation results for the DoE samples, the DKG models are created. The accuracy of the DKG model is checked by comparing the change in the mean squared error of the previous DKG model and the updated DKG model when additional DoE samples are added in the 'sequential DoE sampling' loop seen in Fig. 5. Additional DoE samples are added, i.e. more casting simulations are performed, until the DKG model converges, i.e. the DKG model does not change as more DoEs are added. Once the DKG model has converged to the desired accuracy, Monte Carlo simulations at 200 000 random points are evaluated using the DKG model in each optimisation iteration shown in Fig. 5 as the 'update design' loop. Using the results of the Monte Carlo simulation, the probabilistic sensitivity is calculated using the statistical distributions of the input random variables (i.e. distributions of R and H). Interested readers are referred to Lee *et al.*[15,16] for details on these probabilistic sensitivity calculations, as that is out of the scope of this paper. The probability of failure and sensitivity results from the Monte Carlo simulations are then provided to the optimisation algorithm. The RBDO optimiser used in the I-RBDO software is MATLAB's built in sequential quadratic programming algorithm.

Description of example for casting process feeding system optimisation

The example case study application presented here compares the DDO and RBDO methods for the casting feeding system design as shown in Fig. 1. The casting is a 600 mm long by 100 mm wide by 50 mm thick bar. A cylindrical shaped riser is used and shown in green in the

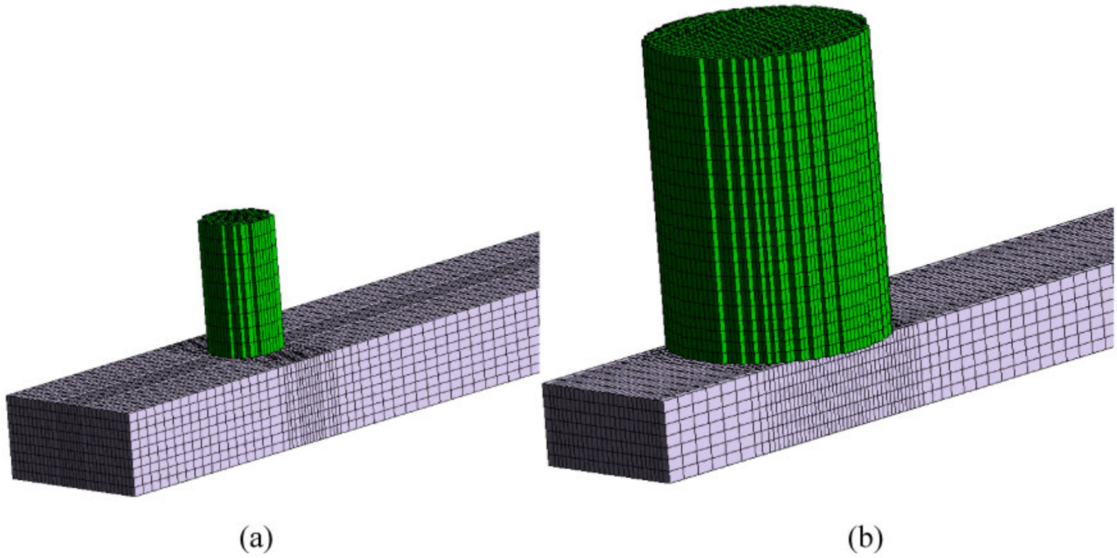


5 Computational flow diagram of sampling based RBDO in the I-RBDO software

figure. The riser volume is to be minimised; this is the cost function. The constraint in this optimisation problem is that the average porosity within the bar casting must be <0·1%. Hence, the casting yield is to be maximised while keeping the average porosity in the bar to be <0·1%. The two design variables to be optimised are the radius *R* and height *H* of the riser, which are considered to include uncertainties arising from the manufacturing process. Since a sampling based optimisation method is used, the sensitivities of the riser volume to the radius and height are not explicitly used by the software. For the RBDO analysis, the uncertainties in *R* and *H* must be defined. In this example problem, it is assumed that there is much less control in the process of filling the riser to a given height than to form the riser radial dimension during the moulding process. The distributions for *R* and *H* to be used in the RBDORBDO analysis are shown schematically in Fig. 1. Here, the uncertainty in *R* is defined by assuming it follows a normal distribution with a standard deviation of 3 mm, and the uncertainty in *H* is defined by a standard deviation of 10 mm. The results from the DDO and RBDO cases will be compared to a case termed here as a 'typical practice' riser design. For the 'typical practice' case, the riser aspect ratio (height divided by diameter) is assumed to be constant at 1, and the smallest riser size was found by trial and error simulations that satisfied a 10 mm safety margin, defined as the distance between the end of the riser pipe and the casting cope surface. Because the 'typical practice' case employs a safety margin, it is also referred as the safety margin method of riser design.

Casting simulations were performed to determine porosity in the casting using *MAGMAsoft* version 4.6 with steel properties for the German steel grade GS-20Mn5 from the software's database. This steel is a 0·2%C, 1·25%Mn plain carbon steel with chemistry similar to an AISI 1522 steel. It has a liquidus temperature of 1506°C and solidus temperature of 1428°C. Filling was not simulated, and an initial temperature of 1600°C was used. The casting software *MAGMAsoft* solves the heat conduction/solidification problem, assuming no melt flow. Its porosity predictions are based on a proprietary model, the accuracy of which cannot be assessed in the current study. If the porosity model's accuracy was known, its statistical variability could be included in the RBDO analysis, but that is outside the scope of the current work. In the casting simulation software, there is an adjustable parameter (feeding effectivity) used in the porosity prediction algorithm, which was set to 30%. Green sand mould properties from the *MAGMAsoft* property database were used. The casting simulation software's automatic mesh generation algorithm was used throughout the optimisation process with a constant number of computation volumes set to 200 000. This algorithm generates the computation meshes for the casting, riser and mould using this value for target total number of computation volumes used in the simulation. The resulting mesh is typical of what a foundry engineer might use in initial and routine simulations to develop a casting production process.

In Fig. 6, the riser end sections of two computational meshes used in the optimisation study are shown with the riser in green and part of the casting in grey. The



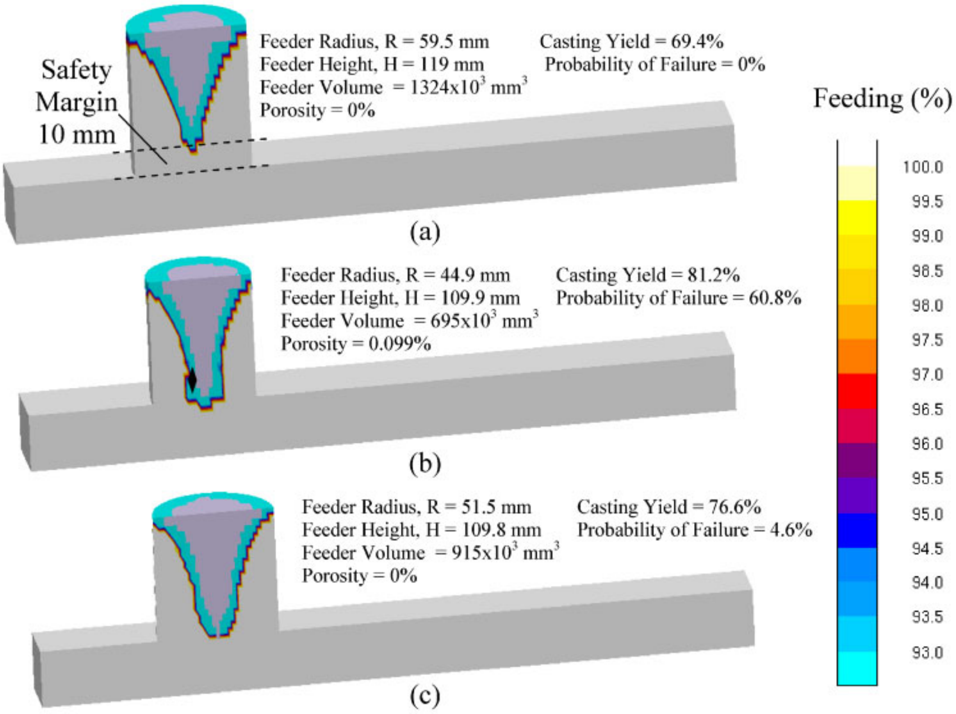
6 Example computational meshes used in casting simulations showing risers (green) and part of casting (grey) used for *a* smallest riser sizes simulated (*R*=20 mm, *H*=80 mm) having 24 192 metal CVs and *b* largest riser size simulated (*R*=65·4 mm, *H*=186 mm) having 37 364 metal CVs

mesh for the smallest riser size simulated (R=20 mm, H=80 mm) is shown in Fig. 6*a*, having 24 192 metal computational volumes (CVs). The mesh for the largest riser size simulated (R=65·4 mm, H=186 mm) having metal 37 364 CVs is shown in Fig. 6*b*. These meshes give relatively fast run times of 1 to 2 min per casting simulation on a Linux workstation using four Intel Xeon E5472 3·0 GHz CPUs. Approximately 360 casting simulations were required in the DDO and RBDO analyses performed for a total simulation run time of ~9 h for this optimisation problem. This demonstrates the practical feasibility of the optimisation method and software since the casting simulations could be run even faster on a newer CPU and a 16 CPU workstation or a cluster.

The 'typical practice' casting simulation cases were run first to determine the smallest riser size that satisfies the margin of safety condition. Once R and H for the 'typical practice' case were found, those dimensions were used as the starting point for the DDO analysis. Before the search ranges for R and H and convergence tolerances were established for the DDO and RBDO solutions, 106 test simulations were run, covering ranges for R and H from 20 to 65 mm for R, and from 66 to 175 mm for H. This testing led to establishing search ranges and convergence tolerances for the I-RBDO method, such that a solution could be achieved in a reasonable number of simulations. Porosity results from these 106 test runs are combined with the 116 casting simulations used in the DDO solution to create a visual map of how the porosity constraint varies with R and H, which will be shown in the results section. After the testing simulations, the search ranges for R and H for the DDO and RBDO analyses presented in this paper were defined in the software as 30 to 65 mm for R, and 60 to 190 mm for H.

The I-RBDO software uses the sequential quadratic programming algorithm in MATLAB for both DDO and RBDO analyses. The MATLAB optimisation algorithm uses normalised tolerances, and these were set to values recommended by the I-RBDO developers. In the DDO analyses, the tolerances for the objective function and variables were set to 0·001, and the constraints to 0·05. In the RBDO analysis, all tolerances were 0·05.

Using the I-RBDO software, the DDO analysis was performed first followed by the RBDO analysis. The commercial casting simulation package *MAGMAsoft* was run in an iterative fashion for the cases requested by the I-RBDO software. All casting simulations and their interfacing with the I-RBDO software were performed manually. As requested from the output of the I-RBDO software, a casting simulation case having a riser size of R and H to be simulated is selected according to its design of experiments algorithm. For a given simulation case, the preprocessor in the *MAGMAsoft* software was used to set up the case having the R and H to be simulated, the mesh generator was executed to prepare the case and then the simulation was executed. After running the case, the post-processor was loaded and was used to determine the average porosity for the casting (excluding riser) as a 'user result'. When the average casting porosity is determined from the casting simulation for a given set of values of R and H, the porosity (i.e. constraint) and riser volume (i.e. cost) are passed back to the I-RBDO software. In other words, the porosity and riser volume are the performance measure responses input to the I-RBDO software at its requested sets of variables R and H. The I-RBDO software analyses this information from the casting simulation as if it were a 'black box' as discussed in connection with Fig. 5. Then, according to its DoE and other algorithms, the I-RBDO tool requests a new casting simulation case (or cases) to be run until the optimisation process has converged. Automating the interfacing of the casting simulation and the I-RBDO software would save significant time required for the overall process. The process of manually using the preprocessor to set up a case, to the extracting of results in the post-processor, and to inputting them to the I-RBDO software required about four additional minutes per simulation beyond the casting simulation time. This was about twice the actual



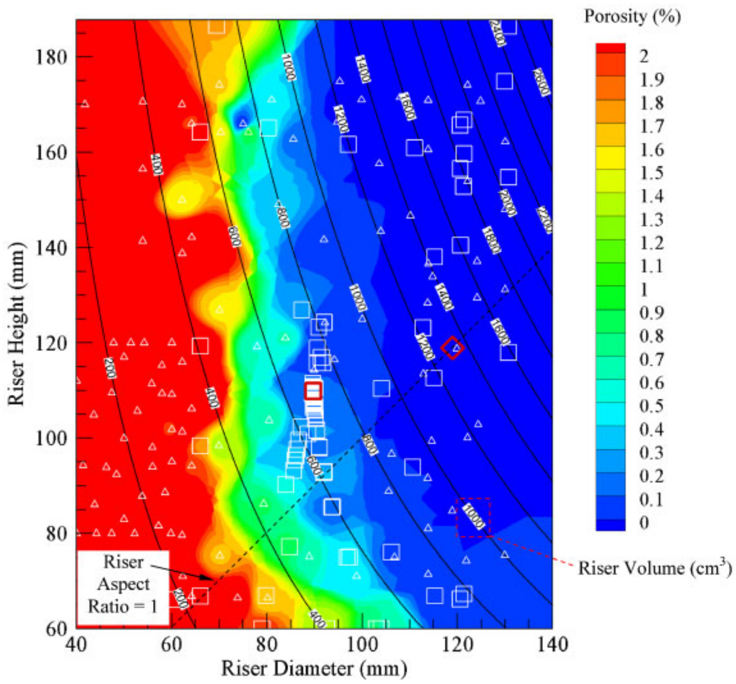
7 Midwidth sections showing riser piping via *MAGMAsoft* feeding percentage result for *a* case run using 10 mm safety margin to determine riser height with riser diameter equal to plate width, *b* resulting riser piping and results for DDO analysis and *c* results and predicted riser piping for RBDO case

casting simulation CPU time. Automating the interfacing of the I-RBDO and casting simulation software used is an ongoing task for the future. After that development, more complex investigations using RBDO to design casting processes will be possible.

Casting process feeding system optimisation results

In Fig. 7, cross-sections are shown through the midwidth section of the casting and riser to visualise the feeding 'shrinkage pipe' for the three approaches used to design the risers. The shrinkage pipe is the light purple (empty) v shaped region in the riser that transitions through the blue (porous) region to the grey (sound) region. This v shaped shrinkage pipe forms as metal is drained from the riser to make up for solidification shrinkage in the casting. In Fig. 7*a*, the result is shown for the 'typical practice' case with a riser aspect ratio of 1 and using a 10 mm safety margin to determine the riser size of 119 mm height and 59·5 mm radius. The resulting riser dimensions, riser volumes, average porosity predicted and casting yield are given for each feeding system design method in Fig. 7.

The dependence of the performance measures (riser volume and porosity) on the variables R and H is shown in Fig. 8. This figure is analogous to Fig. 2, where the general concepts of the DDO solution were presented. This figure visualises the 'typical practice' and DDO design solutions relative to the performance measures and variables as shown by the red diamond (◇) and red square (□) for the 'typical practice' and DDO solutions respectively. Rather than use R, Fig. 8 uses diameter, since the aspect ratio (height divided by diameter) can be more readily determined from the plot. A dashed line is provided to show risers having an aspect ratio of 1, since recommended values for aspect ratios in steel casting are in the range 1 to 1·25. In Fig. 8, the porosity is plotted as the coloured flood contours. The contours are determined from 232 scattered simulation porosity results at values of R and H used in the I-RBDO testing and to obtain the DDO solution. Kriging interpolation was used in the commercial plotting software *Tecplot* to generate the contour map. The 232 porosity results plotted in Fig. 8 are made up of 106 simulations shown by small white triangles (Δ) that were run during the I-RBDO testing and determining good settings to use (i.e. parameters for design variable search ranges and solution tolerance), and the 126 simulations run afterwards for solving the DDO problem, shown as larger white squares (□). The testing simulation run results were used as initial seed data for the DDO solution. Therefore, 232 casting simulation results were provided to the I-RBDO software to determine the DDO solution. Note too that the porosity visualised through the scattered data Kriging interpolation in the plotting software is not equivalent to output from the Kriging surrogate modelling used in the I-RBDO method. The porosity performance measure constraint can be visualised by the contours ~0·1% porosity. The performance measure objective function (riser volume) is shown in Fig. 8 as the black solid contour lines. The sets of diameter and H values requested in the DDO solution process are shown by the white squares in Fig. 8, while the triangles give the initial cases provided to the I-RBDO analysis. For the DDO cases simulated (white squares) note, there are many simulation runs in the neighbourhood of the DDO solution (red square symbol), indicating the tolerances in the I-RBDO analysis could have been relaxed. This is a demonstration of the importance in gaining experience with the I-RBDO method and its parameters in order to solve a



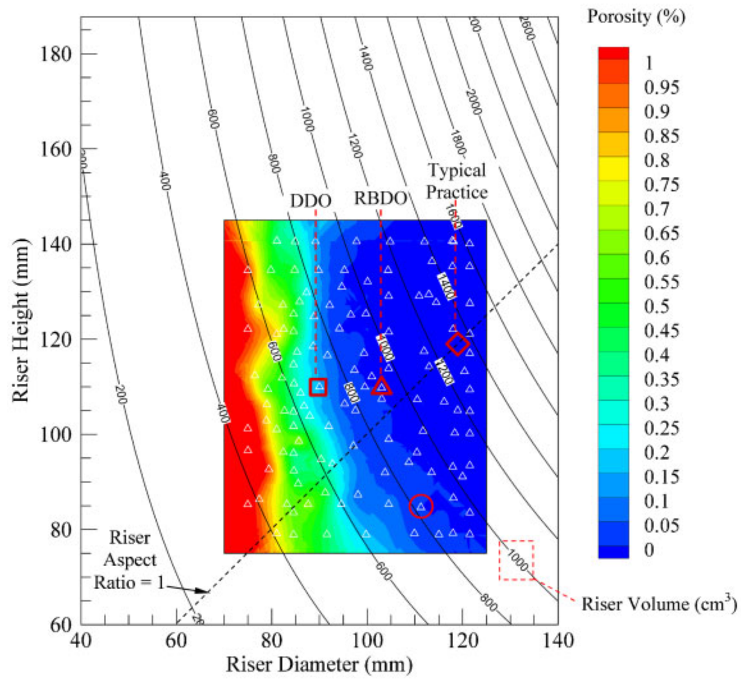
8 Interpolated average porosity at riser diameter and height values from casting simulations for 232 test runs and DDO cases run as coloured flood contours. Small white triangles (Δ) are 106 test simulations run before the DDO simulations. Larger white squares (□) are 116 simulations run to solve DDO solution. Riser volume is given by black solid contour lines. Red symbols are 'typical practice' solution at diamond (◇) and DDO solution at square (□)

problem efficiently. Porosity results are clearly much more sensitive to changes in diameter than height, which is due in part that riser volume changes with the square of changes in R. The quite conservative nature of the 'typical practice' result is apparent from the plot as well. The coloured contour plot of the porosity results in Fig. 8 demonstrates that even for a relatively straightforward casting process design case such as this, the porosity prediction does not have a smooth appearance. The porosity contours have isolated regions of higher and lower porosity, and many more casting simulations would be required to smooth out the porosity plot. This is an indication that this DDO problem is ideally suited to a sampling based optimisation method like that in the present I-RBDO tool.

Similar to the DDO solution visualised in Fig. 8, the corresponding RBDO solution with constraint and objective functions is visualised in Fig. 9 along with the design points for the DDO and safety margin casting process solution methods. In Fig. 9, the 125 casting simulation riser design points that were run in the RBDO solution process are shown using the small white triangles (Δ). Using the scattered data Kriging interpolation in the plotting software, the porosity is plotted using coloured flood contours in Fig. 9 for the RBDO solution casting simulation cases. Here, porosity results are plotted on a more sensitive porosity scale than Fig. 8, since this finer porosity scale (to a maximum of 1%) is well defined for the 125 RBDO casting simulation cases. This is because the RBDO design point values are more densely and uniformly distributed in the search window in Fig. 9 than in Fig. 8. This results from the DoE method used in constructing the surrogate model for the reliability analysis. In Fig. 9, the larger red triangle is the RBDO solution. The other red symbols are the 'typical practice' solution indicated by the diamond (◇) and the DDO solution as the square (□). Here, it is clear that the RBDO solution is achieved by increasing the radial dimension of the riser. The interpolated porosity contours in Fig. 9 suggest a better location for the RBDO solution, indicated by data circled by the red '○', where the porosity is as low as the RBDO solution and the porosity contours forgiving to changes in R and H. In fact, looking through the casting simulation results, there appears to be one result at R=55·6 mm and H=84·7, where the porosity vanishes. However, that result appears to be an outlier and the software clearly rejected the region as feasible for the RBDO solution.

A summary of the results from the casting feeding system design studies for all three design methods is provided in Fig. 10*a* for the riser dimensions and aspect ratios, and in Fig. 10*b* for the casting yield and probability of failure results. The casting yield for the 'typical process' safety margin design approach is ~69%, and its probability of failure was found to be 0% based on the assumed uncertainties in R (standard deviation of ±3 mm) and H (standard deviation of ±10 mm), as seen in Fig. 10*b*. This casting process design approach is conservative.

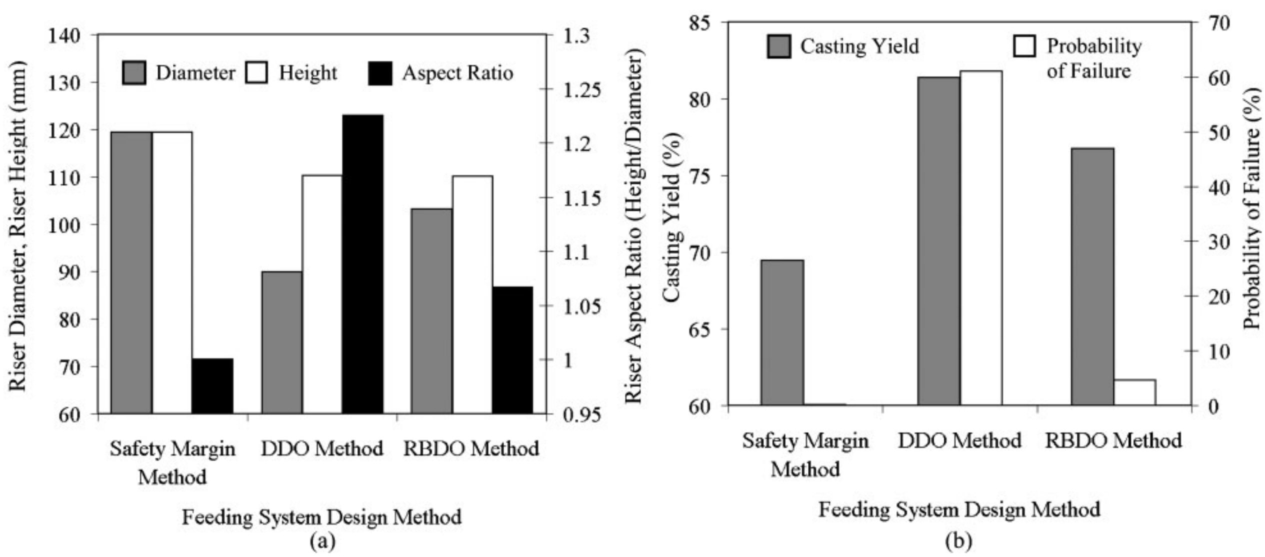
The optimised casting process solution from the DDO method is summarised in Fig. 7*b* and in Fig. 10 by the middle bars. The DDO solution required 116 runs of the casting simulation software to determine the solution. In the DDO result, both R and H are markedly reduced from the safety margin method; R is reduced from 59·5 to 44·9 mm, and H is reduced from 119 to 109·9 mm. The DDO solution maximises the casting yield as seen in Fig. 7*b* while keeping porosity in the casting to just <0·1%. The DDO solution has an average casting porosity of 0·099%, whereas the safety margin method had 0%. The casting yield in the DDO solution is 81·2%,



9 Interpolated average porosity as coloured flood contours at riser diameter and height values from casting simulations run to solve RBDO design problem. Small white triangles (Δ) are 125 points of RBDO casting simulation cases run. Larger red triangle is RBDO solution. Other red symbols are 'typical practice' solution at diamond (◇) and DDO solution at square (□)

about a 12% increase over the safety margin method. Qualitatively speaking, the shrinkage pipe of the DDO solution feeder in Fig. 7*b* appears much flatter at the bottom than that in the safety margin method. It appears the DDO riser piping has no margin for error that it might extend down into the casting and violate the porosity constraint. It is not surprising that the probability of failure for the DDO solution was determined to be 60·8% as seen in Fig. 7*b*.

The RBDO solution riser pipe and results shown in Fig. 7*c* required an additional 125 casting simulations, which is an additional 54% computation time over the DDO solution. A target probability of failure for the RBDO solution of 5% was provided to the I-RBDO software before running this case. The resulting probability of failure of the RBDO solution was 4·6%, which is much lower than that of the DDO solution but higher than that of the safety margin method, which is a very conservative design approach. The safety margin method design was found to have 0% probability of failure because none of the 65 simulation cases in its local window for reliability analysis resulted in any porosity in the casting. The large probability of failure for the DDO solution is not surprising. Deterministic design optimisation solutions are typically found to have a probability of failure in the neighbourhood of 50%



10 Summary results from casting feeding system design studies for three design methods *a* riser dimensions and aspect ratio and *b* casting yield and probability of failure of design to meet porosity constraint

when analysed using the I-RBDO method. The difference in the probability of failure between the RBDO and the safety margin case is insignificant from a practical point of view. For the RBDO solution, the casting yield is 76·6%, which is 4·6% less than the DDO solution, and is 7·2% higher than the safety margin solution as seen in Fig. 7*b*. These results indicate that the 10 mm safety margin design approach gives a conservative, safe design, but it is less economical than the RBDO solution. Clearly, the DDO method offers a dramatic increase in casting yield (or decrease in riser volume), but it is not feasible.

Note that in Figs. 7 and 10*a*, the H for the RBDO solution is nearly identical to that for the DDO solution (109·8 versus 109·9 mm for RBDO and DDO respectively), and R is increased from 44·9 mm in the DDO solution to 51·5 mm in the RBDO solution. This shift in the radial dimension from the DDO to the RBDO solution is also visualised in Fig. 9. The uncertainty for H is much larger than that for R, and to prevent the shrinkage from piping into the bar, one might wrongly think that H should be increased to be sure it is large enough to prevent this by increasing the safety margin. However, many foundries know from experience that the radial dimension of risers should be increased to prevent piping into the casting. The resulting RBDO solution is seen to agree with foundry practice and achieves its solution by increasing R to a large enough value that the solution is insensitive to a large change in H. From the riser volume contour lines and the coloured porosity contour lines in Fig. 9, which run somewhat parallel to increasing H, it is also clear that increasing H in the region of the solution does not give a reduction in porosity, only a moderate increase in riser volume.

For top risers used in steel casting (the type of riser examined here), the aspect ratio [$AR = H/(2R)$] is recommended to be at least 1, and for side risers (with contact to the side rather than the top of a casting) as large as 1·5. If the AR exceeds 1·5, there is no benefit to the riser's feeding effectiveness arising from the additional height and the additional wasted metal in the riser is uneconomical. In addition, secondary under riser shrinkage may form in steel castings with ARs $>1{\cdot}5$. The AR results of this study are shown in Fig. 7*a*. The safety margin approach used an AR of 1, the DDO result gave an AR of 1·22, while the AR for the RBDO solution was 1·07. As a result from the RBDO analysis, ARs closer to 1·1 for casting feeding systems with top risers can be used and appear to be more efficient and reliable.

Conclusions

Application of optimisation methods to casting process design provides more than just optimal solutions. It provides an overview of possible solutions, some of which might be novel and innovative. It gives foundry engineers insight into the sensitivity and stability in both the actual process, and process models, to variables and parameters. Here, RBDO and casting simulation are integrated to go a step further in the development of optimisation methods by including uncertainties in process and model variables, and determining an optimal solution with a known probability of success.

For the typical approach to riser design using a safety margin of 10 mm, the probability of failure was 0% based on assumed uncertainties in riser height and radius and the results of the casting simulations. The probability of failure for the RBDO design method was 4·6%. The safety margin design approach gives an overly conservative safe design that is much less economical than the RBDO solution, which had a 7% casting yield improvement over the safety margin approach. It has been demonstrated that a purely deterministic optimal solution offers a remarkable 12% increase in casting yield over typical design practice, but had an unacceptable 61% probability of failure. The deterministic optimisation solution required 232 casting simulations and the RBDO solution required an additional 125 simulations. This resulted in an additional 54% computation time required in achieving the RBDO solution. It is well worth the additional cost to achieve an optimal solution that is reliable, since the deterministic solution is not. Another advantage of the RBDO method is that its output consists not only of a reliable optimum design but the reliability level of the design. Not only is the RBDO more reliable than the DDO solution, but it has a higher casting yield than the safety margin approach, which is representative of industrial practice. As a result, the additional computational cost of the RBDO method is offset by manufacturing cost savings. An additional insight from this study is that the RBDO solution corresponds to the practice followed by most foundries, that increasing the radial dimension of risers, rather than the height, is the most reliable way to resize a riser that is not feeding a casting adequately.

It has been demonstrated that a deterministic optimisation approach produces a design that is only ~50% reliable and therefore fails to produce a reliably sound casting. The results demonstrate that further research into applying the RBDO method to casting process design and cast component design is worth pursuing. Since the interfacing between the RBDO optimisation and casting software has been manual up to this point, a conclusion of this study is that developing an automated interface between them is a worthy topic for future work. The fact that full automation of casting process design optimisation is possible and effective has been demonstrated for DDO in recent versions of commercially available casting simulation software.[6–8] Such autonomous methods eliminate the need to manually set up each new simulation case (geometry, meshing, etc.) and extract results from the post-processor for input into the optimisation software. While the hundreds of simulations that are necessary in an optimisation may seem like a large number, automation and other recent advances have made computational casting optimisation practical for industry. After this development, follow on investigations building from the current work can examine more complex castings, and more process variables and uncertainties. Numerous foundries already use DDO based casting process design optimisation software[6–12] for castings that are much more complex than the simple example presented here. Since RBDO increases the computation time by only ~50% over DDO, RBDO can be expected to become a valuable tool in the casting industry. In addition, linking mechanical property predictions from casting simulation results to finite element stress analyses along with the RBDO analyses is an continuing topic of research of the authors. This is leading to the development of new methods for reliability based manufacturing process and design optimisation of castings.

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